On System Complexity: Identification, Measurement, and Management

Casti, J.L.

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International Institute for Applied Systems Analysis
A-2361 Laxenburg, Austria

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FOREWORD

This paper represents the written version of a lecture given at the Workshop on Complex Systems in Abisko, Sweden, May 1984. In its current form it will appear as a chapter in the forthcoming IIASA book, *Complexity, Language and Life: Mathematical Approaches*, J. Casti and A. Karlqvist, eds.

Boris Segerstahl
Leader
Science & Technology Program

ABSTRACT

Attempts to axiomatize and formalize system complexity all leave a feeling of basic incompleteness and a sense of failure to grasp important aspects of the problem. This paper examines some of the root causes of these failures and outlines a framework for the consideration of complexity as an implicate, rather than explicate, property of systems in interaction.

On System Complexity: Identification, Measurement, and Management

John L. Casti

Complexity and Simplicity†

> I have yet to see any problem, however complicated, which, when you looked at it the right way, did not become still more complicated.
> *Poul Anderson*

The notion of system complexity is much like St. Augustine's description of time: "What then is time [complexity]? If no one asks me, I know; if I wish to explain it to one that asks, I know not." There seem to be fairly well-developed, intuitive ideas about what constitutes a complex system, but attempts to axiomatize and formalize this sense of the complex all leave a vague, uneasy feeling of basic incompleteness, and a sense of failure to grasp important aspects of the essential nature of the problem. In this chapter we examine some of the root causes of these failures and outline a framework for the consideration of complexity that provides a starting point for the development of operational procedures in the identification, characterization, and management of complex processes. In the process of developing this framework for speculation, it is necessary to consider a variety of system–theoretic concepts closely allied to the notion of complexity: hierarchies, adaptation, bifurcation, self-organization, and reductionism, to name but a few. The picture that emerges is that of complexity as a *latent* or *implicate* property of a system, a property made explicit only through the interaction of the given system with another. Just as in baseball where some pitches are balls and some are strikes, but "they ain't nothin'" until the umpire calls them, complexity cannot be thought of as an intrinsic property of an isolated (closed) system; it is only made manifest by the *interaction* of the system with another, usually in the process of measurement and/or control. In this sense, it is

†Notes and references relevant to each section are given at the end of the chapter.

probably more meaningful to consider complexity more as a property of the interaction than of the system, although it is clearly associated with both. The exploration and exploitation of this observation provides the starting point for an emergent *theory* of complex processes.

Before embarking upon a detailed consideration of complexity in natural and human phenomena, it is useful to consider for a moment why a deeper understanding of complexity, *per se*, is of either theoretical or practical importance. The basic reason is the seemingly inherent human need to simplify in order to understand and direct (control). Since most understanding and virtually all control is based upon a *model* (mental, mathematical, physical, or otherwise) of the system under study, the simplification imperative translates into a desire to obtain an equivalent, but reduced, representation of the original model of the system. This may involve omitting some of the original variables, aggregating others, ignoring weak couplings, regarding slowly changing variables as constants, and a variety of other subterfuges. All of these simplification techniques are aimed at reducing the degrees of freedom that the system has at its disposal to interact with its environment. A theory of system complexity would give us knowledge as to the limitations of the reduction process. For example, it is well known that the three-body problem of celestial mechanics cannot be resolved in analytic terms; however, the two-body problem is completely solvable, but a sequence of two-body problems cannot be combined to solve the three-body problem. Thus, the complexity of the three-body problem is intrinsically greater than any sequence of two-body problems and there is an irretrievable loss of information in passing to such a reduced representation. A useful theory of system complexity would provide conditions under which such a decomposition would work and perhaps even suggest novel, nonphysical, simpler representations that would be valid when the "natural" simplifications fail.

What are the distinguishing structural and behavioral characteristics of those systems we intuitively think of as being complex? Perhaps the easiest way to approach this question is to consider its converse: what features do we associate with *simple* systems? Some of the most evident properties of simple systems are:

- *Predictable behavior*. There are no surprises: simple systems exhibit a behavior pattern that is easy to deduce from knowledge of the external inputs (decisions) acting upon the system. If we drop a stone, it falls; if we stretch a spring and let it go, it oscillates in a fixed pattern; if we put money into a fixed-interest bank account it grows to a sum according to an easily understood and computable rule. Such predictable and intuitively well-understood behavior is characteristic of simple systems.

  Complex processes, on the other hand, display counter-intuitive, seemingly acausal behavior full of unpredictable surprises. Taxes are lowered and unemployment and stagflation persist; low-cost housing projects generate slums worse than those the housing replaced; construction of freeways results in unprecedented traffic jams and increased commuting times. For many people, such unpredictable and seemingly capricious behavior *defines* a complex system.

- *Few interactions and feedback/feedforward loops.* Simple systems generally involve a small number of components, with self-interaction dominating the mutual interaction of the variables. For instance, primitive barter economies involving only a small number of goods (food, tools, weapons, clothing) are generally much simpler and easier to understand than the developed economies of industrialized nations, in which the pathway between raw material inputs and finished consumer goods follows a byzantine route involving large numbers of interactions between various intermediate products, labor, and capital inputs.

  Besides involving only a few variables, simple systems generally have very few feedback/feedforward loops. Such loops enable the system to restructure, or at least modify, the interaction pattern of its variables, thereby opening-up the possibility of a wider range of potential behavior patterns. As an illustration, imagine a large organization characterized by the variables: employment stability, substitution of work by capital, and level of individuality (personal level). Increased substitution of work by capital decreases the human level in the organization, which in turn may decrease employment stability. Such a feedback loop exacerbates any initial internal stresses, potentially leading to a collapse of the process. This type of collapsing loop is especially dangerous for social resilience and is a common feature of complex social phenomena.
- *Centralized decision-making.* Power in simple systems is generally concentrated in one or, at most, a few decision-makers. Political dictatorships, privately owned corporations, and the Roman Catholic Church are good examples of such systems. These systems are simple because there is very little interaction, if any at all, between the lines of command. In addition, the effect of the central authority's decision upon the system is usually rather easy to trace.

  By contrast, complex systems display a diffusion of *real* authority. There is generally a nominal, supreme decision-maker, where the buck stops, but in actuality the power is spread over a decentralized structure, with the actions of a number of units combining to generate the system behavior. Typical examples include democratic governments, labor unions, and universities. Systems exhibiting distributed decision-making tend to be somewhat more resilient and more stable than centralized structures, as they are more forgiving of mistakes by any one decision-maker and are more able to absorb unexpected environmental fluctuations.
- *Decomposable.* Typically, a simple system involves weak interactions among its constituent components. Consequently, if we sever some of these interactions the system behaves more-or-less as before. Relocating American Indians to reservations produced no major effects on the dominant social structure in Arizona, for example, since, for cultural reasons, the Indians were only weakly coupled to the local social fabric. Thus, the simple social interaction pattern could be further decomposed and studied as two independent processes, the Indians and the settlers. A similar situation occurs for the restricted three-body problem, involving the Sun, Earth, and Moon. For some purposes, this system can be decomposed by neglecting the Moon and so studied as a simpler two-body problem.

On the other hand, a complex process is irreducible. Neglecting any part of it or severing any connection usually irretrievably destroys essential aspects of the system's behavior or structure. We have already mentioned the unrestricted three-body problem in this regard. Other examples include the tripartite division of the US government into executive, judicial, and legislative subsystems, an RLC electrical circuit, and a Renoir painting.

The picture that emerges from the foregoing considerations of simple systems is a notion of complex phenomena characterized by counter-intuitive behavioral modes that are unpredictable from knowledge of environmental inputs; by relatively large numbers of variables interacting through a rich network of feedback/feedforward connections; by decentralized decision-making structures and a high level of functional indecomposability. Since such features are characteristic of many of the human systems of modern life, it is necessary to develop effective procedures for managing and planning the future course of such processes. Let us briefly consider some of the issues involved in obtaining a handle on complex systems.

Management of the Complex

> Some problems are just too complicated for rational, logical solutions. They admit of insights, not answers.
> *J. Wiesner*

We have already noted that system complexity is a contingent property arising out of the interaction I between a system S and an observer/decision-maker O. Thus, any perception and measure of complexity is necessarily a function of S, O, and I. Conditioned by the physical sciences, we typically regard S as the active system, with O being a passive observer or disengaged controller. Such a picture misses the crucial point that generally the system S can also be regarded as an observer of O and that the interaction I is a two-way path. In other words, for a given mode of interaction I, the system S displays a certain level of complexity relative to O, while at the same time O has a level of complexity relative to S. For the sake of definitiveness, let us denote the former as *design complexity* and the latter as *control complexity*. It is our contention that the behavior of S becomes uncontrollable when these two complexity levels are too far apart; hence the "golden rule" for management of complex systems is to arrange matters so that

design complexity = control complexity.

The distinction between design and control complexity has been blurred in the natural sciences because of the almost universal adoption of the tacit assumption that the interaction I is one-way, from O to S. When S is a system of macroparticles as in, say, the observation of an oscillating pendulum in mechanics, it is defensible to argue that the pendulum cannot "see" O or, at least, the pendulum has no awareness of O as a system with which it is in interaction. Hence, there is no notion of control complexity and the regulation and management of S by O proceeds according to classical principles. But when we pass to the microscopic and quantum levels or to the global and cosmic levels, the assumption of no control complexity becomes increasingly difficult to defend. And by the time we move to systems possessing even primitive levels of self-awareness in biology and the social sciences, we can no longer neglect the inherent symmetry in the interaction I. The first step in addressing management issues for complex systems is the explicit incorporation of control complexity into the modeling and decision-making framework.

To illustrate the above points, consider the structure associated with representative government at the regional or national level. Here we have a system S composed of the political leaders (mayor, governor, etc.) interacting with a system O consisting of the general public. If the complexity of S as perceived by O is high, then the public sees its leaders as taking incomprehensible actions; they see a byzantine and unwieldy governmental bureaucracy and a large number of independent decision-makers (government agencies) affecting their day-to-day life. In short, what would be observed is exactly what is seen in most countries today. On the other hand, if the political leadership were to perceive the public as being very complex, what would their observations be? They would see a seemingly fickle, capricious public, composed of a large number of independent self-interest groups clamoring for more and more public goods and services. Furthermore, there would be a perception that the public interest groups were connected together in a rather elaborate network that could not be decomposed into simpler subgroups. Consequently, actions or decisions taken to address the interests of one group could not be isolated in their effect, which may possibly be contrary to the interests of another. Or, even worse, because of the dense web of interconnections and feedback loops comprising the public structure, unpredictable and unpleasant side effects may emerge from actions taken to satisfy some subgroups. It goes without saying that these observations form part of the everyday life of most public officials in the western world (and, most likely, the eastern, too).

From the above considerations, we can conclude that the crux of the problem of modern government *versus* its citizenry is that both the public and the governing officials regard each other as complex systems. If either recognized the other as simple, much of the tension and dissatisfaction with contemporary political structures would disappear. The ideal situation would be for each to perceive the other as simple, in which case both parties would be happy. Failing this, simple government with a complex public or complex government with a simple public would at least reduce the difficulties and tensions in one direction, but with possibly increased tensions in the other. Local administration in a small, rural community would be representative of the former, while a political dictatorship of some sort would be typical of the latter situation. Unfortunately, at the regional and national level throughout most of the western world, we have the complex/complex case, which requires a deeper consideration of how each side comes to attach the label "complex" to the other, before the question of complexity management can be meaningfully addressed.

As emphasized earlier, complexity as a system property emerges from the interaction of a given system with another. If a system S can interact with O in a large number of *nonequivalent* ways, then S regards O as complex; conversely, if S has only a small number of modes of interaction with O, then O appears simple. In the governmental context, a dictatorship appears more complex to the public, because the public has many different modes of interaction with the government since, in such situations, most of the agencies of day-to-day life (police, military, communications, transport, agriculture, etc.) are directly in governmental hands. Such centrally planned structures require a high level of control complexity to maintain and are perceived as complex by other systems which have to interact with them.

A system is counted as simple if there are only a small number of non-equivalent ways to interact with it. The pen I used to write this manuscript is a simple system to me. The only mode of interaction with it that I have available is to use it as a writing instrument; however, if I were, say, a chemical engineer, then many more modes become available. I could analyze the plastic compound of which it is made, the composition of chemicals forming the ink, the design of the writing ball at its tip, and so forth. So, for a chemical engineer my ballpoint pen becomes a far more complex object than it is for me.

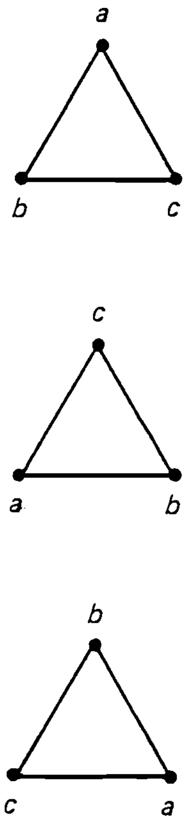
If we adopt the position of this chapter that effective management of complexity consists of arranging systems so that design and control complexity are approximately equal, preferably at a relatively high or low absolute level, then we operationally face the question of how to formally characterize the idea of a system, an interaction between two systems, and the notion of equivalent interactions.

Systems, Observables, and Models

For the things of this world cannot be made known
without a knowledge of mathematics.
Roger Bacon

To progress beyond the obvious and trivial, it is necessary to formalize the common language and linguistic terms used earlier to describe system complexity and its management. Only through such a formalization can we transfer these intuitive, but fuzzy, terms into a mathematical setting that provides the possibility of gaining operational insight into the way complexity is generated and suggests how procedures can be developed to cope with the complex.

For us, a *system* S is composed of an abstract set of states Ω, together with a collection of real-valued *observables* $f_i : \Omega \to R$. For example, let the system S consist of the rotational symmetries of an equilateral triangle. There are then several candidates for the abstract state space Ω, as shown in Figure 6.1. Thus, there is nothing sacred about the state space Ω; it is just a collection of elements that *name*, or *label*, the possible positions of the triangle. A typical observable for this system would be the map f, which assigns to the state $\omega \in \Omega$ the minimal number of rotations through $2\pi/3$ needed to reach ω from the state [a, b, c]. Thus, $f : \Omega \to \{0,1,2\} \subset R$. In this case, if we take $\Omega = \Omega_3$, then $f(\omega) = \omega$, but if we use $\Omega = \Omega_1$ or Ω_2, then $f(\omega) \in \Omega_3$. Consequently, for the observable f it is possible to code any of the states in Ω_2 or Ω_3 by an element of Ω_3; in a certain sense, Ω_3 is a *universal* state space for this system, relative to the observable f.



	Ω_1	Ω_2	Ω_3
=	$[a, b, c]$	0	0
=	$[c, a, b]$	$2\pi/3$	1
=	$[b, c, a]$	$4\pi/3$	2

Figure 6.1

In physics and engineering, it has become common practice to use $\Omega = R^n$ as a universal state space for a system involving n observables, $\{f_i\}_{i=1}^n$. In fact, a good deal of the art behind mathematical modeling in the physical sciences lies in a judicious *choice* of observables $\{f_i\}$, so that the points of R^n serve as a universal coding scheme for the actual abstract states of S. It is both remarkable and unfortunate that this procedure works as well as it does: remarkable since there is no *a priori* reason to expect that the natural world is constructed so as to uniformly lend itself to such an encoding scheme; unfortunate, since the successes in physics and engineering have generated a certain sense of unjustified confidence that a similar procedure will work equally well in the social and behavioral sciences. It does not, which accounts for a great deal of the difficulties found in many attempts to mimic the methods of physics when modeling human affairs. All that having been said, let us return to the formalization of system descriptions and complexity.

From the (possibly infinite) set of all observables characterizing S, we select a subset (usually finite), $F = \{f_1, f_2, \ldots, f_N\}$, and call F an *abstraction* of S. Associated with the abstraction F is a relation, or a set of relations, Φ, between the observables f_i of F,

$$0 = \Phi(f_1, f_2, \ldots, f_N) \ .$$

Such a relationship Φ is termed an *equation of state* or a *description* for the system S. Since the observables are all real-valued functions of Ω, if there are m relations, $\Phi : R^n \to R^m$.

As a simple illustration of the preceding ideas, let the system S be the citizenry of a country. The abstract states Ω of such a system might characterize the political mood of the populace. For this, we could take

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\} \ ,$$

where ω_1 = very content, ω_2 = weakly content, ω_3 = divided, ω_4 = some dissatisfaction, ω_5 = great unrest. Two (of many) observables for this system could be f_1, the fraction of the population favorably disposed to the political party in power, and f_2, the fraction neutral or opposed to the current regime. The actual numerical values of f_1 and f_2 when the system is in any state, $\omega \in \Omega$, need to be determined on empirical grounds. However, we always have the equation of state

$$\Phi(f_1, f_2) = f_1 + f_2 - 1 = 0 \ ,$$

for any $\omega \in \Omega$.

In the above situation, there is no notion of causality. The observables of F and the equation of state Φ are simply quantities that represent our view of the system S; they compactly summarize our experimental and observational knowledge of S; that is, the data. The common manner in which a causal structure is imposed upon the observables is through the recognition that in all systems there are noticeably different time-scales according to which the values of the observables change. We can employ (tacitly or directly) these time-scales to induce a notion of order, or a causal structure, upon F.

To see how a causal structure can be introduced, imagine a system S characterized by an abstraction $F = \{f_1, \ldots, f_N\}$ involving N observables. Further, assume that observation has shown that the observables change on three time-scales, slow, medium, and fast, for example. For the sake of exposition, let the observables be labeled so that

$$a = \{f_1, \ldots, f_k\} = \text{slow} \ ,$$

$$u = \{f_{k+1}, \ldots, f_q\} = \text{medium} \ ,$$

$$y = \{f_{q+1}, \ldots, f_N\} = \text{fast} \ .$$

Let A, U, and Y represent the range of values of the observables a, u, and y, respectively. By the preceding argument, we have $A \subset R^k$, $U \subset R^n$, and $Y \subset R^m$, where $n = q - k$ and $m = n - q$. The causal relationship is induced by invoking the principle that slow dynamics force, or cause, fast dynamics. Thus, we regard a and u as causing y. In common parlance, the slow variables a are generally termed *parameters*, while the medium-speed, causal variables u are termed *inputs* (*controls, decisions*). The response variables y are the system *outputs*.

Usually, there is a feedback effect in that u, and sometimes a, is modified by the output y. But the important point here is that when we think of some observables causing others, it is the rate-of-change of the observables that produces the temporal ordering which we assign to the system. Thus, causality is not necessarily a natural or *intrinsic* aspect of S, but rather is introduced by the way the observer perceives the various time-scales at work in the system. In the classical physical sciences, this point is not usually particularly important and becomes significant only at cosmic and quantum levels; however, in the social and behavioral sciences it is an issue at the very outset, and partially accounts for the difficulties in economic and social modeling of deciding what causes what, a question which lies at the heart of any sort of predictive modeling.

A better intuitive understanding of the partitioning of the system observables is obtained if we employ an evolutionary metaphor. The slow variables α can be thought of as specifying the system genotype; that is, the aspects of S that enable us to recognize the system as S and not some other system S'. For instance, in an urban environment, α might code information about the local geographic, cultural, political, and economic structure that allows us to know we are in Omsk rather than Tomsk. The medium-speed observables u correspond to the system's *environment*. Thus, u represents either natural environmental factors or those created by decision-makers. Finally, the outputs y characterize the morphostructure, or form, of S, the so-called system *phenotype*. For many social systems, y represents the behavioral responses of S to genetic mutation (change of α) and/or environmental fluctuation (change of u). In the urban context, u may reflect various actions by policymakers, such as imposition of zoning restrictions, urban renewal legislation, and the like, while y would then display the effects of those environmental decisions, together with the given genotype (city), as new housing developments, modifications of transport channels, redistribution of industry, and so forth. The important point is the relative time-scales of the processes.

Now let us turn to the central question of this section: how to decide whether two descriptions, or models, of the same system are equivalent. In the above terminology, we have the description

$$\Phi_\alpha : U \to Y \ ,$$

and the description

$$\hat{\Phi}_{\hat{\alpha}} : U \to Y \ ,$$

both purporting to describe the same system S, and our question is whether the two descriptions convey the same information about S or, what amounts to the same thing, do Φ and $\hat{\Phi}$ provide independent descriptions of S?

Mathematically, the descriptions Φ_α and $\hat{\Phi}_{\hat{\alpha}}$ are *equivalent* if there exist maps g and h, depending on α and $\hat{\alpha}$, such that the following diagram commutes:

$$\begin{tikzcd} U \arrow[r, "\Phi_a"] \arrow[d, "g_{a,\hat{a}}"'] & Y \arrow[d, "h_{a,\hat{a}}"] \\ U \arrow[r, "\hat{\Phi}_{\hat{a}}"'] & Y \end{tikzcd}$$

The existence, properties, and construction of the maps $g_{\alpha,\hat{\alpha}}$ and $h_{\alpha,\hat{\alpha}}$ depend strongly upon the mathematical structure assumed for the sets U and Y and the descriptions Φ_α and $\hat{\Phi}_{\hat{\alpha}}$. We do not discuss these matters here. A purely mathematical treatment of the above question forms the core of singularity theory, which is covered in detail by Golubitsky and Guillemin (1973), Lu (1976), and Gibson (1979). The systems view of singularity theory as outlined above is treated in Casti (1984).

It is worthwhile to pursue, for a moment, the implications of system equivalence. If Φ_α and $\hat{\Phi}_{\hat{\alpha}}$ are equivalent, it means that a change of the parameter α to $\hat{\alpha}$ can be neutralized, or cancelled out, by a corresponding relabeling of the elements of the sets U and Y. Speaking metaphorically, if we regard S as an organism described by Φ_α, then the genetic mutation $\alpha \to \hat{\alpha}$ can be made invisible by an appropriate modification of the environment U and the phenotype Y. When put in such terms, the notion of system equivalence is strongly reminiscent of the theory of biological transformations originally developed by d'Arcy Thompson in the early 1900s. In that theory, an attempt was made to show that a common genetic structure in the past could be inferred from phenotypic equivalence in the present. In other words, two species $(y, \hat{y})$ with different genotypes $(\alpha \neq \hat{\alpha})$ in the *present*, would be considered to have arisen from a common ancestor $(\alpha = \hat{\alpha})$ in the *past*, if there is a phenotypic transformation h which transforms one species into the other. This is clearly a special case of our diagram when the environment U is held fixed (g = identity).

For given genotypes α and $\hat{\alpha}$, it may be that there exist no transformations g and h which enable us to pass from Φ_α to $\hat{\Phi}_{\hat{\alpha}}$. In this case, there exist mutations $\hat{\alpha}$ near α that result in qualitatively different phenotypic structures. Such a situation forms the underlying basis for a theory of *bifurcation* and *catastrophes*, which we consider in more detail below.

The Emergence of Complexity

> The electron is not as simple as it looks.
> *Sir William Bragg*

The complexity of a system S is a contingent property, depending upon the nature of the observables describing S, the observables characterizing the system O measuring S, and their mutual interactions. Imagine that O sees S in an operational mode which O describes by the equation of state Φ_α. Further, suppose that at another time O sees S in the mode $\hat{\Phi}_{\hat{\alpha}}$. If Φ_α and $\hat{\Phi}_{\hat{\alpha}}$ are equivalent, in the sense described above, O concludes that S is manifesting essentially the same behavior in the two modes, and O is able to use equally well either description to characterize *both* modes of S. On the other hand, if $\Phi_\alpha \not\sim \hat{\Phi}_{\hat{\alpha}}$ (i.e., they are not equivalent), O is unable to reduce one description to the other and regards the operation of S as being more *complex*, since O sees more variety in the possible modes of S's behavior. This simple idea forms the nucleus of our main thesis that

complexity of S	=	the number of nonequivalent descriptions
(relative to O)		Φ_α that O can generate for S.

Interchanging the roles of S and O, the complexity of O relative to S is defined in a similar manner. Let us denote these two complexities as $C_O(S)$ and $C_S(O)$, respectively. Thus, $C_O(S)$ is what we earlier termed design complexity, while $C_S(O)$ is the control complexity of the joint system S and O.

A crucial aspect of our notion of system complexity is that it is a ***comparative*** concept: there is a tacit assumption that in order to compute $C_O(S)$, O must have available a ***family of descriptions*** of S and a method for deciding whether or not two descriptions from the family are equivalent. If Q denotes the family of descriptions, the above procedure defines an equivalence relation on Q, thereby partitioning it into appropriate equivalence classes. Since, by definition, all descriptions belonging to a given class are equivalent, the number $C_O(S)$ is just equal to the number of classes that Q is separated into by our concept of system equivalence. To operationally implement this procedure, the following steps are needed:

(1) Beginning with a fixed description S construct a ***family*** Q of descriptions containing S as a member. One fairly standard way of doing this has already been described above, when we begin with the description $\Phi(f_1,\ldots,f_N)$ and isolate some observables as parameters $\boldsymbol{\alpha}$. The values of $\boldsymbol{\alpha}$ then provide a parameterized family of descriptions of S.

(2) Partition Q into equivalence classes in accordance with the equivalence relation "$\sim$" described earlier. To accomplish this task, it is necessary to employ the machinery of singularity theory, once the mathematical character of Q and the equivalence relation are fixed.

(3) Calculate $C_O(S)$ = card $Q/\sim$ = the number of classes into which Q is split by the relation $\sim$.

In terms of management and decision-making, it is O who must select the family Q and the relation $\sim$; different selections lead to different levels of complexity as perceived by O. Similar remarks apply to the view of O as seen by S.

A simple example in which the above concepts are explicitly displayed is when $\Phi : U \to Y$ is linear with $U = R^n, Y = R^m$. In this case, Φ can be represented by an $m \times n$ matrix, once bases are chosen in U and Y. In order to parameterize the description Φ, let us suppose that we regard the first diagonal element of Φ as a parameter; that is $\alpha = [\Phi]_{11}$. Then the family $Q = \{\Phi_\alpha : R^n \to R^m, \alpha \in R\}$. Now let P and Q be linear coordinate transformations in U and Y, respectively, and suppose we consider an alternative description, $\Phi_{\hat{\alpha}}$; that is, we change the value of the element $[\Phi]_{11}$ from α to $\hat{\alpha}$. We ask if $\Phi_\alpha \sim \Phi_{\hat{\alpha}}$ or, what is the same thing, does the diagram

$$\begin{tikzcd} U \arrow[r, "\Phi_a"] \arrow[d, "P"'] & Y \arrow[d, "Q"] \\ U \arrow[r, "\Phi_{\hat{a}}"'] & Y \end{tikzcd}$$

commute? Well-known results from matrix theory tell us that in this case $\Phi_\alpha \sim \Phi_{\hat{\alpha}}$ if and only if

$$\text{rank } \Phi_\alpha = \text{rank } \Phi_{\hat{\alpha}} \ .$$

Thus, if we let $\alpha = \min \{m, n\}$, we can assert that

$$\text{complexity } \Phi_\alpha \leq \alpha + 1 \ .$$

The exact complexity, of course, depends upon the structure of the fixed elements of Φ_α. If, for example, rank Φ_α is constant for all $\alpha \in R$, then complexity $\Phi_\alpha = 1$. Thus,

$$\text{complexity } \Phi_\alpha = \text{number of different values that rank } \Phi_\alpha \text{ assumes as } \alpha \text{ ranges over } R.$$

In passing, we note that the points $\alpha^* \in R$ at which Φ_α changes rank are what we earlier termed bifurcation points. They represent places where the inherent information in the description Φ_α (here represented by the number of linearly independent rows of Φ_α, for example) is different from that in Φ_{α^*} for α near α^*. We return to this point in a more general context later.

In summary, complexity emerges from simplicity when alternative descriptions of a system are not reducible to each other. For a given observer, the more such inequivalent descriptions he or she generates, the more complex the system appears. Conversely, a complex system can be simplified in one of two ways: reduce the number of potential descriptions (by restricting the observer's means of interaction with the system) and/or use a coarser notion of system equivalence, thus reducing the number of equivalence classes. The first strategy is exemplified by a decision-maker who listens to only a few advisors before making a decision rather than gathering a full spectrum of views on a particular issue; a failure to dig deep enough to get all the facts surrounding a situation before taking action would be representative of the second approach to simplification. Both approaches are considered in more detail below, but first let us examine some of the ways in which the complexity of a system can change in a *natural* manner.

The Evolution of Complexity

> In short, the notion of structure is comprised of three key ideas: the idea of wholeness, the idea of transformation, and the idea of self-regulation.
>
> *J. Piaget*

It has been recognized, at least since the work of Turing and von Neumann on self-reproducing machines, that in order for a system to evolve to a higher level of complexity, it is necessary for the system to contain its own self-description. We might well ask why it would not be possible to design a self-reproducing system with given functional characteristics using hardware alone, without also requiring an internal linguistic description of what it is doing. The answer lies in the conditions for reliability, adaptation, growth, and evolution that we use to characterize complex systems; we are not interested in a system whose natural tendency is to degenerate or lose its function. Systems that contain their own genetic description are one known type of organization that allows survival and evolution despite errors within the system, or even errors in the description. In general, we have only a feeble understanding of the explicit conditions for the linguistic descriptions needed to achieve the threshold of reliability and adaptability necessary for survival and evolution.

In the above view, a complex system is a composite consisting of a physical structure (the hardware) carrying out functions under the instructions of an internal description of itself (the software). This situation would be well understood, as it is in computer science, if it were not for the fact that in most systems of interest the hardware and software are contained in the same physical structure. A key problem in the understanding of complex processes is the way in which the dynamic modes of the system interact with the linguistic modes, and the manner in which these complementary modes are combined to provide an external observer with some level of complexity, as outlined earlier. If we regard a *measurement* process as a physical structure that executes a rule which relates a system to an element of its description, then the encoding of dynamical processes to linguistic structures is very closely related to measurement. On the other hand, the decoding and physical execution of a genetic description is a problem of *interpretation*.

The measurement/interpretation complementarity can be very easily demonstrated by examining ordinary human speech. We can either say what we mean or we can examine how we have said it, but we can't do both simultaneously. We can represent physical structures as descriptions only when we recognize that the structures are obeying a coherent set of rules, which we call a language. And it is in this language that we formulate our concepts of complexity or simplicity. The irony in this picture is that the natural language we use to identify complexity may cause us to interpret inherently simple events, as seen by the internal language of our self-describing system, as complex messages in our interpretative natural language. An important component in the management of complexity is the institution of procedures to bring the internal and natural languages much closer, and so to prevent the external observer from receiving a message that is not really in the system itself.

Considerations of structure and description also bear heavily upon the emergent complexity arising out of lower level, simpler processes. If we think of the evolutionary process, in general, as a mapping of environmental variety and constraints into the structure of the evolving system in the form of organizing principles and coded information, then it is possible to distinguish three quite distinct evolutionary strategies: the *phylogenetic*, *ontogenetic*, and *sociogenic*. Let us consider these strategies in light of our earlier remarks.

- *Phylogenetic*. This strategy involves random genetic mutations and gene mixing which are tested in their phenotypic forms by interaction with environmental stresses. The successful structures (if any) result in the blind, natural selection of the corresponding genotypes. In terms of our earlier formalism, the map $\Phi_\alpha : U \to Y$ is modified by purely random changes in α with future mutations of α entirely unaffected by the resulting phenotypes $y_\alpha(u)$. Such a strategy is enormously profligate and slow, permitting rapid environmental fluctuations to reduce the viability of species before the phylogenetic mapping can catch up as, for example, with the extinction of the dinosaurs.

- *Ontogenetic.* If the system has some means of storing the results of mutations in α, for example, with some neurophysiological structure like a brain, then instead of random genetic changes, we have selective trial-and-error probings of the environment. In short, the genetic changes are directed by what has gone before in a process called *learning*. Such an ontogenetic strategy permits a more rapid and refined process of information generation about the environment; there is an adaptive mechanism by which successful phenotypic characteristics are fed back to the gene pool to promote further genotypic changes. We might think of this feedback or learning mechanism as embodied in the neural code of the system, as opposed to its genetic code. However, this strategy also has its drawbacks, principally the fact that the information is stored in the system and goes out of existence with its death.
- *Sociogenic.* This strategy is associated with systems that are not only social, as in various insect societies, but also sociocultural, which involves not only a permanent social organization, but also an arbitrary symbolic coding of the role relationships in the society. At this level, the sociogenic strategy of evolution involves an additional code, the *normative* code, which is stored outside the physical system itself. Thus, the information about the environment does not die with the system and, in fact, can be passed on to new systems without their having to first directly experience the actual environmental interactions. In this strategy, besides the advantage of extra-somatic storage of information, there is the possibility of the system restructuring itself very rapidly when environmental pressures become great enough.

In the sociogenic strategy, we pass from a variation of the genetic code to mutations of the normative code, which guides the social and psychological development of new generations. Instead of a gene pool comprising the system's stock of coded information, there is an idea pool which is a reservoir of the culture's templates for the coordination and integration of individual actions and interactions. New ideas or ideologies are continually generated as mutations, subject to various selection pressures, with reproductive success measured by the perpetuation of one normative system and social structure as opposed to others.

As a simple illustration of sociocultural evolution, consider the development of societal regulatory mechanisms; that is, the dominant political structures. The appearance of democratic forms of social regulation represents, from the purely objective point of view of cybernetics, the evolution of a more adaptive political structure. For example, a more extensive idea pool, fuller information and feedback channels in the system, and a more extensive mapping of the internal as well as external states of the system and environment.

Of special importance is the balance between those institutional structures and processes designed to maintain a given structure and those designed to enable better adaptation to environmental conditions. The former structures are much more strongly incorporated into the micro- and macrostructure of the political system than the latter; hence, pressures tend to mount until the old structure can be changed only through potentially destructive revolution – a singularly poor strategy for evoluton.

Our previous consideration of system complexity as a property of the interaction between a system and its observer/regulator applies at each level of the above evolutionary scheme. However, we can also think of the emergence of a new ***type*** of system complexity as we pass from the phylogenetic to sociogenic strategies. This is an evolution not of the complexity displayed by a fixed system, but rather a qualitative change of the type of system from individual, nonlearning units to social collections of adaptive units, each system type requiring its own complexity concept. We touch on some of these distinctions in the next section which deals with the interrelationships between system complexity and the concepts of adaptation, hierarchy, and bifurcation.

Complex Systems: Adaptation, Hierarchy, and Bifurcation

> There is nothing in the whole world that is permanent. Everything flows onward; all things are brought into being with a changing nature; the ages themselves glide by in constant movement.
> *Ovid* (Metamorphoses)

Treatments of complexity often place great emphasis upon various behavioral or structural characteristics of a system, which, if present, offer supposed *prima facie* evidence that the system is complex, by whatever interpretation the author is advocating. Three of the most commonly cited characteristics are:

- *Adaptability.* The capacity for the system to monitor its environment and to reconfigure itself on the basis of its observations in order to more effectively perform its function.
- *Hierarchy.* The tendency for the system to be structurally organized in a stratified manner so that information and activities at lower levels are combined as inputs to higher levels, while overall direction and control passes from higher to lower levels.
- *Bifurcation and novelty.* The tendency for complex processes to spontaneously display a shift from one behavioral or structural mode to another, as levels of organization increase. These surprises or emergent novelties represent points of bifurcation where a previous description of the system breaks down and a new description, not reducible to the old, is required.

While it should be clear by now that we do not hold to the view that any of the above features is an infallible indicator of complexity, it certainly is true that many complex phenomena *are* hierarchically structured, *do* display emergent behavioral modes, and *can* adapt to new situations. Consequently, it is of interest to examine how well these system properties can be accommodated to the complexity concept introduced earlier in this chapter.

Adaptation

Consider the capability of a system to *adapt* to changing conditions in the environment. This is a functional concept involving at least some subsystems changing their functional behavior to accommodate the new environment. A political system granting voting rights to women in response to egalitarian social currents, as in Switzerland in recent times, is the type of adaptive change a complex system can often make. So is the way in which banks have been introduced into modern economic structures as an adaptation to provide for intertemporal exchanges in disequilibrium. Here, a subsystem whose previous function was only to act as a storehouse of wealth, has changed its function to provide credit and other services which allow an economy to sustain a continual state of disequilibrium. One might say, even, that all adaptation arises as a result of a principle of function change, whereby subsystems created for one function begin to perform a quite different function when the system perceives the new function to be evolutionarily more advantageous than the old. The classical biological example of this kind of shift is the evolution of the human eye, which cannot confer any survival advantage until it sees and cannot see until it is highly evolved and complex. Thus, it is difficult to imagine how such an organ could arise as the result of minute differential changes in a fixed organ, even over millions of years. It is much more reasonable to suppose that originally the eye performed a function quite different from sight and an accidental feature of this proto-eye was that it was photosensitive. As time wore on, the photosensitivity feature became more and more evolutionarily advantageous and the original function of the eye was lost.

The picture of adaptation as being a system response to changed circumstances leads to the basic evolutionary equation

variation + selection = adaptation,

expressing the fact that, in order to adapt, the system must have many potential modes of behavior and a procedure for evaluating the relative fitness of the various alternatives in a given environment. One of the difficulties with complex human social systems is that redundancy at the genetic level, which gives the capacity for independent variations, is too limited. As a result, there is too little room for trying new approaches and for exploring alternative pathways to a given functional goal when operating circumstances change. Systems such as large nuclear power plants, national economies, major ecosystems, and the like have little, if any, degrees of freedom in their structure or design with which to experiment. The consequences of a failure are too great to allow the evolutionary equation to operate effectively, at least in its natural mode. In our view, until more resilient design policies are employed for such large-scale systems, the only possible way to escape this prison of hypotheticality is by way of mathematical models and computer exploration of alternative systems, rather than by relying upon nature's trial-and-error. On balance this is probably a better strategy anyway, since we don't have millions or even hundreds of years to find solutions to our energy, economic, and environmental problems. But the potential Achilles heel in the computer simulation strategy is that it is totally dependent upon the existence of faithful models of reality, expressible in mathematical terms. Thus, the weight of the entire edifice is concentrated upon the need to develop a science of modeling and effective procedures for the identification of "good" models of human and natural phenomena.

To incorporate the above ideas into our earlier formalism, we must introduce a feedback mechanism through which environmental fluctuations are sensed by the system and used to generate exploratory variations in the system's "genomes". Recalling that the basic description (or model) of the system is given by a family of relations

$$\Phi_\alpha : U \to Y \ ,$$

inclusion of adaptive capabilities requires two steps:

(1) *Feedback/feedforward loops.* The system genome α is now thought of as being at least partially determined by either current and past states of the environment (feedback), in which case $\alpha = \alpha[u(t - \tau)]$ and/or upon predicted future states of the environment (feedforward). In the latter event, $\alpha = \alpha[\hat{u}(t + \tau)]$. Here τ is some time-lag, while $\hat{u}$ denotes the predicted future environmental state. There are good arguments for both feedback and feedforward mechanisms in adaptive structures and, most likely, any truly self-organizing complex structure develops both modes for coping with environmental change.

(2) *Selection procedure.* Implicit in the above feedback/feedforward mechanism is a selection procedure; the environment is sensed and predicted and a rule is applied which tells the system how to modify its genome to best fit the changed circumstances. Thus, the feedback/feedforward loops represent both random and directed search in the space of the genomes, together with a procedure to weed out the "good" genetic patterns from the "bad".

At this point it is useful to note the distinction between the adaptive capability of an individual system and the effect that the association of individuals in a society has on this capacity. Basically, the adaptive capacity of an individual is reduced, but group adaptive capacity is increased as individuals join together in cellular societies. The key point here is that the group capacity is increased, but on a much longer time-scale than that for individuals. Thus, individual companies join together to form a multinational conglomerate, thereby gaining a group ability to respond to global economic fluctuations that no individual member could easily accommodate, but on a much longer time-scale than the reaction time of a typical firm. It is probably fair to say that higher-level associations only arise through defects in the adaptive capability of individuals. More than any other factor, it is this limited adaptive capacity of individuals that gives rise to the hierarchical organizations so typically present in complex systems.

Hierarchy

The failure of individual subsystems to be sufficiently adaptive to changing environments results in the subsystems forming a collective association that, *as a unit*, is better able to function in new circumstances. Formation of such an association is a *structural* change; the behavioral role of the new conglomerate is a *functional* change; both types of change are characteristic of the formation of hierarchies. It has been argued by Simon (1969, 1981), as well as others, that evolution favors those systems that display stable, intermediate *levels of structure*. Furthermore, a complex system is incomprehensible unless we can simplify it by using alternative *levels of description*. A digital computer illustrates both types of hierarchies, where we have structural or hardware levels from microchips to functional units like disc drives, terminals, processors, and so on. On the descriptive side, we have the system software which describes what the structural levels are to do, using a series of descriptive levels from machine languages to high-level, natural-language programming languages.

In a hierarchical structure, the various levels of organization refer primarily to different ways in which it is possible for us to interact with the system, i.e. nonequivalent types of state descriptions generate different hierarchical levels. It is not possible, for instance, to understand the machine language operations represented by a particular BASIC statement without moving away from the level of BASIC to the more microscopic level of machine instructions. The two descriptions are incompatible in much the same way that it is impossible to understand a biological organism by studying its individual atoms and molecules. Of course, the same situations occur repeatedly in economics under the rubric micro–macro problems, as well as in urban studies, psychology, sociology, and many other areas.

It is interesting to note that in hierarchical organizations, the organizational characteristics *look the same* at each level, in that the dynamics and structural interactions at each level appear to be models of each other. This feature was noted long ago by Haeckel in his bioenergetic law – "ontogeny recapitulates phylogeny", expressing the observation that each organism carries the entire history of the phylum within itself. Other examples of this principle abound: computer programs and their subroutines, a symphony and its various movements, a neural network and the associated network of genetic control, a book and its component chapters, and so on. Some of these hierarchies are structural, while others are functional, and it appears safe to say that the central problem of hierarchy theory is the understanding of the relation between the structural and the descriptive (or functional) levels. Most of the classical physical sciences have concentrated upon structural decompositons, culminating in today's multimillion-dollar searches for the ultimate particles of matter. This is suitable for the study of physics, but for an understanding of living systems (biological, human, social) it is necessary to look for *functional* decompositions: the new reductionism will be based upon units of function and description, not units of structure.

How can the preceding concepts of hierarchical levels be incorporated into our mathematical formulation? At the structural level, the atoms of our modeling formalism are the real-valued observables $f_i : \Omega \to R$, where Ω is the system's set of abstract states. In a loose sense, $\{f_i\}$ are the state variables of the model. Structural hierarchies are formed by combining these state variables, either by aggregation or disaggregation, into new quantities. Imagine that we have n observables that can be collectively written $f = (f_1, \ldots, f_n)$. A hierarchy is formed by prescribing a rule for combining these quantities into m new observables, $\hat{f} = \{\hat{f}_1, \hat{f}_2, \ldots, \hat{f}_m\}$; that is each $\hat{f}_i = \hat{f}_i\,(f_1, \ldots, f_n)$. Diagrammatically, we have

$$
\begin{tikzcd}
\Omega \arrow[r, "f"] \arrow[d, "\alpha"'] & R \arrow[d, "\beta"] \\
\hat{\Omega} \arrow[r, "\hat{f}"'] & R
\end{tikzcd}
$$

in which, the map β is either an imbedding or a projection of $R^n \to R^m$, depending upon whether $n < m$ or $n > m$. The interesting part of the diagram involves the map α and the new state space $\hat{\Omega}$. Since $(\hat{\Omega}, \hat{f})$ represent a different hierarchical level than (Ω, f), it is generally the case that $\Omega \neq \hat{\Omega}$; that is, the set of states appropriate for characterizing the system at a given level is not generally the state set appropriate for another level. But the diagram makes it clear that there is some flexibility in passing from Ω to $\hat{\Omega}$. We can either choose α, thereby fixing the new state set $\hat{\Omega}$, or we can choose $\hat{\Omega}$ and then determine α from the relation $\beta \circ f = \hat{f} \circ \alpha$. The picture sketched above provides a prototypical framework for all structural stratifications that involve the introduction of hierarchies through aggregation and disaggregation.

The descriptive stratification proceeds on the basis that the system activity is determined by the equation of state that links its observables. Thus, the function that the system performs is described by the rule

$$\Phi(f_1, \ldots, f_n) = 0 \ .$$

Earlier, we subdivided the observables using cause-and-effect arguments and wrote this relationship as

$$\Phi_a : U \to Y \ .$$

Now let us consider what is implied when the system passes to a new descriptive level at which a new function is performed. In our context it can mean only one thing: the equation of state Φ has been modified to a new equation $\hat{\Phi}$, possibly (but not necessarily) with a change of observables from $f \to \hat{f}$; that is, in diagrammatic form

$$\begin{tikzcd} U \arrow[r, "\Phi_a"] \arrow[d, "g"'] & Y \arrow[d, "h"] \\ U \arrow[r, "\hat{\Phi}_{\hat{a}}"'] & Y \end{tikzcd}$$

We have already discussed the ramifications of this diagram and note here only that the appearance of a new functional hierarchical level is abstractly the same as the occurrence of a bifurcation in the system description. Consequently, the emergence of new functional hierarchies is completely intertwined with the concept of system bifurcation, and an understanding of the system's functional levels of organization can only occur through a deeper investigation of the number and type of its bifurcation points.

Bifurcation, Error, and Surprise

Earlier, we considered the situation in which there were two descriptions of a given system, say Φ_a and $\hat{\Phi}_{\hat{a}}$, and addressed the question of when we could meaningfully say that Φ_a was *equivalent* to $\hat{\Phi}_{\hat{a}}$. It was argued that $\Phi_a \sim \hat{\Phi}_{\hat{a}}$ if maps g and h could be found such that the diagram above commutes. In other words,

$\Phi_a \sim \hat{\Phi}_{\hat{a}}$ if a change of genotype $a \to \hat{a}$ can be neutralized by appropriate changes, g and h, of the environment and phenotype, respectively. If no such g and h exist (within some appropriate class of maps), then a is called a *bifurcation point* for the description Φ (or, equivalently, $\hat{a}$ is a bifurcation point for the description $\hat{\Phi}$). We then *define* the complexity of the system in terms of the number of bifurcation points. So in this sense, a system S is more complex than a system S' if our description of S contains more bifurcation points than our description for S'. Thus, the concept of system complexity and the idea of a bifurcation are intimately linked at the very outset of our theory: increased complexity can only emerge at a bifurcation point and, conversely, every bifurcation point gives rise to a new mode of system behavior that is not reducible (i.e. understandable) in terms of the old. Now let us consider a quite different way in which bifurcations can generate emergent behavior when two systems are made to interact with each other.

Consider the simple situation in which we have Ω = the real numbers R, and the observables $f = (f_1, \ldots, f_n)$, are defined as

$$\begin{aligned} f_i : R &\to R \qquad\qquad\qquad\qquad i = 1,2,\ldots,n. \\ r &\to i\text{th coefficient in the} \\ &\quad\ \text{decimal expansion of } r \end{aligned}$$

Then, clearly, $r_1, r_2 \in R$ are equivalent with respect to the observables f when r_1 and r_2 agree in the first n terms of their decimal expansions. Now choose numbers r_1', r_2' such that

$$r_1 \sim_f r_1' \quad , \quad r_2 \sim_f r_2' \quad .$$

Now we let the 1-system interact with the 2-system through multiplication; that is, we form the products $r_3 = (r_1 r_2)$ and $r_3' = (r_1' r_2')$ and find that, in general, $r_3 \not\sim_f r_3'$; that is, the equivalence classes under f are split by the interaction (i.e., by the dynamics). In other words, the interaction generates a bifurcation of the f-classes, a bifurcation that we usually call round-off *error*, in the above context. It is instructive to examine the source of this so-called error.

To see the way the error is introduced in the above situation, let us consider a numerical example. Let $r_1 = 123$, $r_1' = 124$, $r_2 = 234$, and $r_2' = 235$, and use $f = (f_1, f_2)$; that is, the equivalence relation generated by f is such that two numbers are equivalent if they agree in the first two places. Here we have $r_1 r_2$ $(= 28782) \not\sim_f r_1' r_2'$ $(= 29140)$, a discrepancy with our expectation based on the f-equivalence. Our surprise at finding $r_1 r_2 \not\sim_f r_1' r_2'$ occurs because the set of observables $f = (f_1, f_2)$ is too limited, thereby causing an unrealistic expectation concerning the interaction between the 1- and 2-systems. If we had expanded the set of observables to the set $\hat{f} = (f_1, f_2, f_3)$, then no such discrepancy would have occurred, since there would be no equivalence, at all, of r_1, r_1' under $\hat{f}$. So, the entire source of our observed error is purely from the incompleteness in the description of the system.

The preceding arguments are entirely general: error (or surprise) always involves a discrepancy between the objects (systems) *open* to interaction and the abstractions (models, descriptions) *closed* to those same interactions. The remedy is equally clear, in principle: just supplement the description by adding more observables to account for the unmodeled interactions. In this sense, error and surprise are indistinguishable from bifurcations. A particular description is inadequate to account for uncontrollable variability in equivalent states and we need a new description to remove the error.

It is interesting to note that since bifurcation and error/surprise are identical concepts, and that complexity arises as a result of potential for bifurcation, we must conclude that complexity implies surprise and error; that is, to say a system displays counter-intuitive behavior is the same as saying that the system has the capacity for making errors, although the error is not *intrinsic* to an isolated system, but occurs when a system interacts with another.

Models, Complexity, and Management

The man who draws up a program for the future is a reactionary.
Karl Marx

It has been said that the reason we construct models is to be able to say "because". Coping with complexity involves the creation of faithful models of not only the system to be managed, but also of the management system itself. As we have continually emphasized, complexity, its identification and control, is an interactive concept between the system and its manager and it is impossible for the management system to effectively regulate the controlled system without having a concept (read: model) of itself, as well as of the system to be managed. This self-description is essential if the management system is to survive in the face of inevitable error and environmental disturbances of the type discussed above. In our earlier terms, effective complexity management reduces to the simple prescription

design complexity = control complexity.

But, what is involved in reaching this state of system–theoretic nirvana?

One aspect we can be certain of is that the search for effective management of complexity does not necessarily involve simplifying the process to be regulated. As Einstein pointed out, things should be as simple as possible, but no simpler, which we could translate as reducing the design complexity to the level of the control complexity, but no lower. Turning this argument around, we can also think of *increasing* the complexity of the management system to bring it into line with the design complexity of the system. Thus, effective complexity management may involve *either* simplifying or complexifying, depending upon the circumstances. But, in either case, it is first necessary to have means for assessing the levels of complexity of the two interacting systems. Thus, we must begin to develop the framework for a *theory of models*, one that includes effective methods for identifying the complexity of interacting systems and the means by which the conflicting complexity levels can be brought into harmonious balance.

Imagine, for a moment, that such a theory of models already exists and consider the types of mangement strategies that would serve to balance design and control complexities at some acceptably high level. First, we note that it is not sufficient simply to equalize the complexity levels of the system and its observer/controller. They must be balanced at a sufficiently high level: if I simplify a Chopin piano sonata by requiring that it be played only on the white keys, I have certainly reduced its complexity level to the level of my observational ability (complexity) to understand the piece. However, I obtain very little pleasure from this kind of complexity balance; the variety that makes the piece interesting has been destroyed and I would probably benefit more with no system at all to observe. In this situation, it is far more reasonable to raise the complexity level of my observing system to match the level of the piece, which presumably already exists at a high enough level to perform its intended function. So, any management scheme must begin by taking into account the *absolute* level at which the design and control complexities are to be equalized.

In terms of general control strategies, there are two complementary approaches. One is to develop bifurcation-free and bifurcation-generating *feedback* policies. As has been noted elsewhere, feedback laws have the effect of changing the internal structure of the system they regulate. Of course, in our context this means that any feedback policy has the potential to change the design complexity of the controlled system. Some illustrations of how this can be done are discussed in Casti (1980), although from the somewhat different perspective of optimal control theory, not the more general setting discussed here.

From a management point of view, there are some disadvantages to using feedback policies, the principle one being that any error-actuated feedback law does not even begin to act until the system is already out of control; that is, if there is no error, the system is not being regulated at all. For many engineering systems this situation is quite satisfactory, but in social and behavioral processes we cannot usually be so sanguine about error-actuated control. Generally, in such systems we would like to *anticipate* difficulties and take action now to avoid projected malfunctions later. In human systems, we cannot afford the luxury of waiting for the system to fail before we take remedial action. This basic principle leads to the idea of *anticipatory* control and *feedforward* policies.

The most important feature of anticipatory control systems is that the manager must have a model of the system to be regulated, and his or her actions are dictated by the *regularities* between the behavior of the system, as predicted by the model (which is run on a time-scale faster than real-time), and the actual, observed system behavior at the future time of the model prediction. The prediction and observation are then correlated and the model recalibrated, leading to the idea of adaptive control. Surprisingly, there seems to have been very little study of such processes, although some recent work by Rosen (1979, 1984) promises to redress this imbalance of knowledge between feedback and feedforward regulators.

From the above, the broad outline of a research program for complexity management begins to emerge, and consists of the following major components:

(1) *A Theory of Models*. There is a need for development of a sufficiently rich theoretical framework for mathematically representing processes in the social, behavioral, and cultural environment. This theory must of necessity include methods for identifying relevant observables, state spaces, and equations of state, as well as provide a basis for formally incorporating the complexity, adaptation, hierarchy, and emergence concepts discussed above.

(2) *Anticipatory Control.* A deep investigation into the nature of feedforward policies as opposed to feedback is needed, in order to provide the means for balancing complexity levels between the manager/decision-maker/ observer and the system under consideration. Such an investigation will include studies of adaptive mechanisms, as well as the role of anticipatory policies in reducing/generating bifurcations in the managed system descriptions.

Each of these points need considerable elaboration before they can constitute a plan for a truly creative research program. But already it is clear that creative research is what is needed if any progress at all is to be made in the complexity management problem. And here the emphasis is on the word creative: no pedestrian, pull-the-pieces-off-the-shelf-and-put-them-together type of program will suffice. New ideas and new approaches are the only currency of this realm. It seems appropriate to close by stating a few general features that serve to identify what we mean by creative research, as opposed to the pedestrian. Our advice to anyone contemplating creative research is to:

- Avoid the research literature.
- Avoid practitioner's problems.
- Never put high hopes on any study for any useful information.
- Never plan – especially not in the long term.
- Never apply for a research grant.
- Never give up if everyone thinks you are wrong.
- Give up immediately when they think you are right.

As Nietzsche said, "that which needs to be proved cannot be worth much", so in today's world I won't hold my breath waiting for any putative "research" organizations to adopt even one of the foregoing principles as part of their official posture and manifesto. Nonetheless, the closer an individual researcher comes to adherence to these guidelines, the closer he or she will be to a position from which to crack the nut of system complexity and its management.

Notes and references

Complexity and simplicity

A detailed consideration of the contention that system complexity necessarily relates to the interaction of a given system with its observer/describer/controller is found in

Phillips, W. and Thorson, S. (1975) Complexity and policy planning, in *Systems Thinking and the Quality of Life, Proc. Soc. for General Systems Research Annual Meeting.*

This paper is notable for its review of various concepts of complexity in the field of social system management and for its conclusion that " ... no adequate characterization of the complexity of a system can be given without specifying the class of observers dealing with the system, as well as the specific purposes of the observers". The author's arguments supporting this view of complexity culminate in the contention that "whichever approach we take to modeling the outer environment – the policy problem – the complexity characteristic of the system is contingent upon our description of the relations between the inner environment and the outer. It is a function of the theories we bring to bear upon problems and the way we view the environment".

Some further views along the same lines were expressed by one of the cybernetics pioneers, W. Ross Ashby, in

Ashby, W.R. (1973) Some peculiarities of complex systems. *Cybernetics Medicine* 9: 1–8.

In this paper, Ashby remarked that "a system's complexity is purely relative to a given observer; I reject the attempt to measure an absolute, or intrinsic, complexity; but this acceptance of complexity as something in the eye of the beholder is, in my opinion, the only workable way of measuring complexity".

Basically, the same points have been emphasized in the philosophy of science literature from a somewhat more fundamental perspective; see, for example,

Quine, W.v.O. (1964) On simple theories of a complex world, in J. Gregg and F. Harris (Eds) *Form and Strategy in Science* (Dordrecht: Reidel),

Wimsatt, W. (1972) Complexity and organization, in K. Schaffner and R. Cohen (Eds) *Studies in the Philosophy of Sciences, Vol. XX* (Reidel, Boston),

and the classic paper

Simon, H., (1969) The architecture of complexity, in *Sciences of the Artificial* (Cambridge, MA: MIT Press).

Management of the complex

The concepts of design and control complexity were introduced by Gottinger in the somewhat different context of an automata–theoretic treatment of complexity. For a recent account of his ideas see

Gottinger, H. (1983) *Coping with Complexity* (Dordrecht: Reidel).

This work represents an approach to the problem of system complexity originally initiated by John Rhodes in

Rhodes, J. (1971) *Application of Automata Theory and Algebra* (Berkeley, CA: Lecture Notes, Department of Mathematics, University of California).

The importance of the symmetry of the interaction between the system and its observer/controller has been particularly emphasized in

Rosen, R. (1984) *Anticipatory Systems* (London: Pergamon),

and

Rosen, R. (1978) *Fundamentals of Measurement and Representation of Natural Systems* (New York: Elsevier).

For a discussion of some of the important matters arising from the interactions present in the political process see

Kirby, M.J.L. (1980) *Reflections on Management of Government Within a Democratic Society in the 1980s, Parts I & II.* (Ottawa: Plaunt Lectures, Carlton University).

Works emphasizing similar aspects of complexity in social and behavioral areas include

> Winthrop, H. (1972) Social systems and social complexity in relation to interdisciplinary policymaking and planning. *Policy Sciences* 3: 405–420,
>
> Winham, G. (1976) Complexity in international negotiation, in D. Druckman (Ed) *Negotiations* (Beverly Hills: Sage Publ. Co.),

as well as the Phillips and Thorson article cited earlier.

Systems, observables, and models

A thorough exposition of the ideas surrounding observables, abstractions, and equations of state is found in the Rosen books cited earlier.

The fast–slow distinction as a means of inducing causality is a special case of hierarchical ordering, but in time rather than space. For a discussion of this crucial point, see the book

> Fraser, J.T. (1978) *Time as Conflict* (Basel: Birkhäuser).

Additional discussion of the macro–micro problem is found in

> Allen, T.F.H. and Starr, T. (1982) *Hierarchy* (Chicago: University of Chicago Press).

Use of an evolutionary metaphor to characterize human systems is far from new, dating back at least to Herbert Spencer and the social Darwinists. A modern attempt to mimic biology as a guide to social development is

> Corning, P. (1983) *The Synergism Hypothesis* (New York: McGraw-Hill).

In the economic area, the evolutionary metaphor has been quite well-developed in

> Nelson, R. and Winter, S. (1982) *An Evolutionary Theory of Economic Change* (Cambridge, MA: Harvard University Press),
>
> Boulding, K. (1981) *Evolutionary Economics* (Beverly Hills: Sage Publ.).

Singularity theory is treated from a mathematical point of view in

> Golubitsky, M. and Guillemin, V. (1973) *Stable Mappings and their Singularities* (New York: Springer),
>
> Lu, Y.C. (1976) *Singularity Theory* (New York: Springer),
>
> Gibson, C. (1979) *Singular Points of Smooth Mappings* (London: Pitman).

The connection between these mathematical results and the theory of equivalent systems is made in

> Casti, J. (1984) *System Similarity and Laws of Nature* IIASA WP-84-1 (Laxenburg, Austria: International Institute for Applied Systems Analysis).

The emergence of complexity

For a discussion of the interrelationship between the idea of system complexity as presented here, and the concepts of system error and entropy, see Chapter 5 in Rosen (1978), cited earlier.

Many attempts have been made to define the complexity of a system in terms of properties of the system alone, such as number of components, density of internal interactions, and so forth. Some machine–theoretic efforts along these lines are

Bremermann, H. (1974) Complexity of automata, brains and behavior, in S. Levin (Ed) *Lecture Notes in Biomathematics*, Vol. 4 (Berlin: Springer),

Bremermann, H. (1974) Algorithms, complexity, transcomputability, and the analysis of systems, in W. Reidel, W. Händler, and M. Spreng (Eds) *Proc. Fifth Congress of the Deutsche Gesellschaft für Kybernetik* (Munich: Oldenbourg),

Gaines, B. (1976) On the complexity of causal models. *IEEE Tran. Syst. Man & Cyber.* SMC-6: 56–59,

George, L. (1977) Tests for system complexity. *Int. J. Gen. Syst.* 3: 253–258.

In addition to missing the crucial point that complexity depends upon the *interaction* of a system with another rather than upon the system itself, an annoying aspect of such studies is the way in which the extremely useful term complexity has been usurped by the computer-orientation of such authors and taken to mean something very specific in the context of machines and algorithms. This situation is by no means new, as the computer industry has a long and deplorable history of taking useful terms and concepts, such as information, system, and systems analyst, and then warping the terms to such an extent that their original meanings are totally lost. Normally this distorting process could be dismissed with a casual shrug, as is done in mathematics, for instance, but for the fact that the computer-industry propaganda machines effectively promote their new meaning of these terms to the general public, thereby creating considerable confusion as to the more general, and far more useful interpretations of these important concepts.

A fascinating article involving the use of complexity in assessing aesthetic experience is

Goguen, J. (1977) Complexity of hierarchically organized systems and the structure of musical experiences. *Int. J. Gen. Syst.* 3: 233–251.

This article introduces the concept of *conditional* complexity, based upon past experiences and expectations, and then applies the idea to develop a theory of surprise for musical compositions. For purposes of aesthetic satisfaction, the author concludes that if the conditional complexity of a piece is too low, then our expectations are too easily and too often fulfilled to maintain our interest, whereas if the conditional complexity is too high, our expectations are too often frustrated to permit much listening satisfaction. This argument leads to an aesthetic law of the mean for musical complexity.

The evolution of complexity

System complexity depends upon whether the system is regarded as an object or as a description, a theme explored in detail in

Löfgren, L. (1977) Complexity of systems: a foundational study. *Int. J. Gen. Syst.* 3: 197–214.

The stability and evolutionary potential of self-describing complex systems depends also upon the complementary relation between the dynamic (structural) and linguistic (functional) modes of system description. This relationship is inextricably intertwined with the epistemological problem of measurement. For a detailed consideration of these matters, see

Pattee, H. (1977) Dynamic and linguistic modes of complex systems. *Int. J. Gen. Syst.* 3: 259–266.

A discussion of the several types of evolutionary strategies is found in

Buckley, W. (1977) Sociocultural systems and the challenge of sociobiology, in H. Haken (Ed) *Synergetics: a Workshop* (Berlin: Springer).

Complex systems: adaptation, hierarchy, and bifurcation

A detailed exploration of biological adaptation as a metaphor for human systems is given in

Rosen, R. (1975) Biological systems as paradigms for adaptation, in R. Day (Ed) *Adaptive Economic Models* (New York: Academic Press).

Rather thorough expositions of the nature of adaptive mechanisms in both engineering and living systems are found in the works

Holland, J. (1975) *Adaptation in Natural and Artificial Systems* (Ann Arbor: University of Michigan Press),

Conrad, M. (1983) *Adaptability: The Significance of Variability from Molecule to Ecosystem* (New York: Plenum).

The initial steps toward a theory of anticipatory control involving feedforward loops are outlined in

Rosen, R. (1978) On anticipatory systems: I & II. *J. Social & Biol. Structures* 1: 155–180.

It is worthwhile to note that the formalism for anticipatory control is in the same spirit as the so-called bounded rationality models in economics. See, for example,

Day, R.H. (1985) Disequilibrium economic dynamics: a post-schumpeterian contribution. *J. Econ. Behavior and Org.* (to be published in 1985),

and

Simon, H.A. (1981) *The Sciences of the Artificial* (2nd edn) (Cambridge, MA: MIT Press).

The appearance of hierarchical organizational structures in natural, as well as man-made systems is discussed from several viewpoints in

H. Pattee (Ed) (1973) *Hierarchy Theory* (New York: Braziller).

See also

Jantsch, E. (1980) *The Self-Organizing Universe* (Oxford: Pergamon),

as well as the Allen and Starr book cited earlier.

The emergence of new structures and behavioral modes through parameter fluctuations and environmental variability is discussed in some detail in

Prigogine, I., Allen, P., and Herman, R. (1977) Long term trends and the evolution of complexity, in E. Laszlo and J. Bierman (Eds) *Goals in a Global Community* (New York: Pergamon),

Prigogine, I. (1980) *From Being to Becoming: Time and Complexity in the Physical Sciences* (San Francisco: Freeman).

The concept of surprise as a system bifurcation is explored in

Casti, J. (1982) Topological methods for social and behavioral sciences. *Int. J. Gen. Syst.* 8: 187–210.

A nontechnical consideration of the same circle of ideas and their applied significance is considered in

Holling, C.S. (Fall 1983) Surprise? *IIASA Options* (Laxenburg, Austria: International Institute for Applied Systems Analysis).

A formal theory of surprises, using ideas from algebraic topology, is put forth in

Atkin, R.H. (1981) A theory of surprises. *Environment & Planning B*, 8: 359–365.

While Atkin's theory does not explicitly employ the idea of a system bifurcation, the concept is implicit in his work and a mathematical unification of the two approaches would be a valuable exercise, shedding additonal light on the essential aspects of a nonprobabilistic theory of surprises.

Models, complexity, and management

The question of complexity management is hardly a new one. A nontechnical introduction to some of the important managerial issues that arise is

Beer, S. (1970) Managing modern complexity. *Futures* 2: 245–257.

It is often held that the objective of system management is to stabilize a process in the face of a fluctuating environment and, in this context, that stability and complexity are positively correlated. Discussions of the pros and cons of this dubious argument are found in

Chadwick, G.F. (1977) The limits of the plannable: stability and complexity in planning and planned systems. *Environment and Planning A* 9: 1189–1192,

Pimm, S. (1984) The complexity and stability of ecosystems. *Nature* 307: 321–326.

The question of bifurcation-free feedback control laws is taken up in

Casti, J. (1980) Bifurcations, catastrophes and optimal control, *IEEE Tran. Auto. Control*, AC-25: 1008–1011.

For a discussion of how linear *feedback* control laws alter internal system structure, see

Casti, J. (1977) *Dynamical Systems and their Application: Linear Theory* (New York: Academic Press).

The connection between feedback and feedforward control laws and the effect that each type has on the alteration of system structure is pursued in

Kalman, R. (1971) Kronecker invariants and feedback, in L. Weiss (Ed) *Ordinary Differential Equations* (New York: Academic Press).

The problems of anticipatory control are developed in

Rosen, R. (1979) Anticipatory systems in retrospect and prospect. *General Systems* 24: 11–23.

See also the Rosen works cited earlier.